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Analysis of Conventional and Reflective Butler Matrices with Imperfect Components

J. P. SHELTON AND J. K. HSIAO

Target Characteristics Branch
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In a reflective Butler matrix the input ports are coincident with the output ports. Hence, all error components produced by imperfect hybrid couplers accumulate at the single set of input/output ports, rather than being distributed at the separate sets of input ports and output ports as in the conventional Butler matrix. This report gives a precise scattering analysis of both conventional and reflective Butler matrices made up of imperfect hybrid couplers, and the effect on the performance of such matrices is computed. A computer program for carrying out the analysis is also given.

CONTENTS

ANALYSIS OF CONVENTIONAL AND REFLECTIVE BUTLER MATRICES WITH IMPERFECT COMPONENTS

INTRODUCTION

A Butler matrix that forms a cluster of beams evenly distributed in the sin-θ space is not usually symmetric with respect to a plane midway between the input and output ports. However, by properly adjusting the phase shifts and interconnections one may modify conventional Butler matrix to be symmetric. Such a matrix may also be folded on itself on the line of symmetry, so that the input and output ports are identical. Such a network not only reduces the number of components required; it also becomes a reflection-type system in which the feed positions are in the plane of the aperture. The synthesis of this network was described previously [1,2]. In this report, we analyze the performance of both conventional and reflective Butler matrices. In particular, we investigate the effect of reflected waves on the beam-forming performance. In a conventional Butler matrix, since the input and output ports are separate, the reflected waves emerging from the input ports have no effect on the beam-forming performance. Multiply reflected waves may emerge from output ports; however, their amplitudes are generally small, and their effects are relatively insignificant. In a reflective Butler matrix, the reflected waves accumulate at the input/output ports; hence, the aperture distribution at the antenna array is significantly modified, and this may degrade the beam-forming performance. These effects are investigated, and computer simulated results are presented together with a listing of the computer program.

SCATTERING MATRIX OF A 3-dB HYBRID COUPLER

The basic building block of a Butler matrix is a 3-dB hybrid coupler. For the ideal hybrid coupler, energy fed into any one of the input ports will be split into two equal components, one with a phase shift of 90° relative to the other. However, practical hybrid couplers will in general exhibit amplitude and phase errors in their transfer coefficients. These amplitude and phase errors will affect the transfer coefficients of both reflective and conventional Butler matrices in the same way. That is, the errors in the overall network input/output transfer coefficients will be the same for both conventional and reflective networks. Practical hybrid couplers will also have nonzero reflection and transfer coefficients to the isolated port. For the conventional network, to a first order, the error components due to these effects will appear at the network inputs. For the reflective network, with its inputs and outputs sharing a single set of ports, all error components affect the input/output transfer coefficients.

Thus, the two types of hybrid coupler errors are forward and reverse. Our investigation will be concentrated on the reverse-error components, and we shall assume that there

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is no amplitude or phase error in the forward-transfer coefficients of the 3-dB coupler. The following analysis is based on the assumption that, when an incident wave of unit amplitude is applied to one of the input ports, two waves of amplitude α will emerge from the two output ports, one with a 90° phase shift and the other with no phase shift. Similarly, waves of amplitude β will be reflected to the two input ports. As shown in Fig. 1(a), when an incident wave of unit amplitude is applied at port 12, reflected waves of $-\beta$ and $-j\beta$ appear at ports 11 and 12 respectively and waves of $-j\alpha$ and α appear at ports 21 and 22. For conservation of energy, one has

$$2\alpha^2 + 2\beta^2 = 1. \tag{1}$$

The isolation factor is defined as the power ratio of the reflected wave to the incident wave. In this case, the isolation is

$$I = \beta^2. \tag{2}$$

Accordingly, in terms of the isolation factor,

$$\alpha = \sqrt{0.5 - I}\ . \tag{3}$$

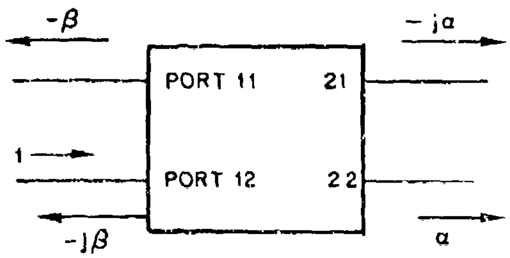
If the parameters in Fig. 1(b) are used, the reflected waves are related to the incident waves by the matrix equation

$$\begin{bmatrix} b_{11} \\ b_{12} \\ \hline b_{21} \\ b_{22} \end{bmatrix} = \left[\begin{array}{cc|cc} -j\beta & -\beta & \alpha & -j\alpha \\ -\beta & -j\beta & -j\alpha & \alpha \\ \hline \alpha & -j\alpha & -j\beta & \beta \\ -j\alpha & \alpha & -\beta & -j\beta \end{array}\right] \begin{bmatrix} a_{11} \\ a_{12} \\ \hline a_{21} \\ a_{22} \end{bmatrix}, \tag{4}$$

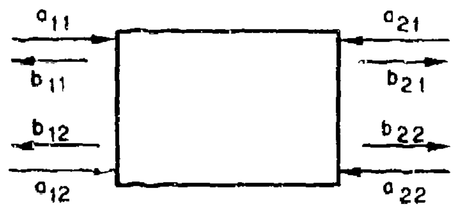
where a_{11}, a_{12}, a_{21}, and a_{22} are incident waves and b_{11}, b_{12}, b_{21}, and b_{22} are scattered waves at ports 11, 12, 21, and 22 respectively.

Let

$$\mathbf{b}_1 = \begin{bmatrix} b_{11} \\ b_{12} \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} b_{21} \\ b_{22} \end{bmatrix},$$
$$\mathbf{a}_1 = \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix}, \tag{5a}$$



(a) Unit-amplitude wave incident at one of the input ports of a 3-dB coupler



(b) Incident and reflected waves on a 3-dB coupler

Fig. 1 — Transfer and reflection in four-port networks

and

$$S_{11} = S_{22} = \begin{bmatrix} -j\beta & -\beta \\ -\beta & -j\beta \end{bmatrix}, \qquad (5b)$$

$$S_{12} = S_{21} = \begin{bmatrix} \alpha & -j\alpha \\ -j\alpha & \alpha \end{bmatrix}.$$

Matrix Eq. (4) can now be simplified to the form

$$\begin{bmatrix} b_1 \\ \hline b_2 \end{bmatrix} = \left[\begin{array}{c|c} S_{11} & S_{12} \\ \hline S_{21} & S_{22} \end{array}\right] \begin{bmatrix} a_1 \\ \hline a_2 \end{bmatrix}. \qquad (6)$$

SCATTERING AND TRANSFER MATRICES OF A BUTLER NETWORK

A Butler network can be represented by a block diagram as shown in Fig. 2.* Blocks in regions 1 and 3 represent the 3-dB couplers described in the previous section, and a phase-shift transfer network is located in region 2. A number of similar networks are

*For the remainder of this report, a network will be considered a physical entity and a matrix a mathematical entity.

connected in cascade to form a complete conventional Butler network. The scattering matrix for regions 1 and 3 is

$$\begin{bmatrix} b_{11} \\ b_{12} \\ \cdot \\ \cdot \\ \cdot \\ b_{1n} \\ b_{21} \\ b_{22} \\ \cdot \\ \cdot \\ \cdot \\ b_{2n} \end{bmatrix} = \begin{bmatrix} -j\beta & -\beta & 0 & 0 & \ldots & \alpha & -j\alpha & 0 & 0 & \ldots 0 \\ -\beta & -j\beta & 0 & 0 & \ldots & -j\alpha & \alpha & 0 & 0 & \ldots 0 \\ 0 & 0 & -j\beta & -\beta & 0\ 0 & \ldots & \alpha & -j\alpha & 0 \ldots & \\ 0 & 0 & -\beta & -j\beta & 0\ 0 & \ldots & -j\alpha & \alpha & 0 \ldots & \\ & & & & & & & & & \\ \ldots & & \ldots & & \ldots & \ldots & & \ldots & & \ldots \\ \alpha & -j\alpha & 0 & 0 & \ldots & -j\beta & -\beta & 0 & 0 & \ldots 0 \\ -j\alpha & \alpha & 0 & 0 & \ldots & -\beta & -j\beta & 0 & 0 & \ldots 0 \\ 0 & 0 & \alpha & -j\alpha & 0\ 0 & \ldots & j\beta & -\beta & & \ldots 0 \\ 0 & 0 & -j\alpha & \alpha & \ldots & \ldots & -\beta & -j\beta & & \ldots 0 \\ & & & & & & & & & \\ \ldots & & \ldots & & \ldots & \ldots & & \ldots & & \ldots \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ \cdot \\ \cdot \\ \cdot \\ a_{1n} \\ a_{21} \\ a_{22} \\ \cdot \\ \cdot \\ \cdot \\ a_{2n} \end{bmatrix} . \tag{7}$$

Define

$$\mathbf{b}_1 = \begin{bmatrix} b_{11} \\ b_{12} \\ \cdot \\ \cdot \\ \cdot \\ b_{1n} \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} b_{21} \\ b_{22} \\ \cdot \\ \cdot \\ \cdot \\ b_{2n} \end{bmatrix}, \tag{8a}$$

$$\mathbf{a}_1 = \begin{bmatrix} a_{11} \\ a_{12} \\ \cdot \\ \cdot \\ \cdot \\ a_{1n} \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} a_{21} \\ a_{22} \\ \cdot \\ \cdot \\ \cdot \\ a_{2n} \end{bmatrix}, \tag{8b}$$

$$S_{11} = S_{22} = \begin{bmatrix} -j\beta & -\beta & 0 & 0 & \ldots & \ldots \\ -\beta & -j\beta & 0 & 0 & \ldots & \ldots \\ 0 & 0 & -j\beta & -\beta & 0\ 0 & \ldots \\ 0 & 0 & -\beta & -j\beta & 0\ 0 & \ldots \\ \ldots & & \ldots & & \ldots & \ldots \\ 0 & 0 & \ldots & \ldots & -j\beta & -\beta \\ 0 & 0 & \ldots & \ldots & -\beta & -j\beta \end{bmatrix}, \tag{8c}$$

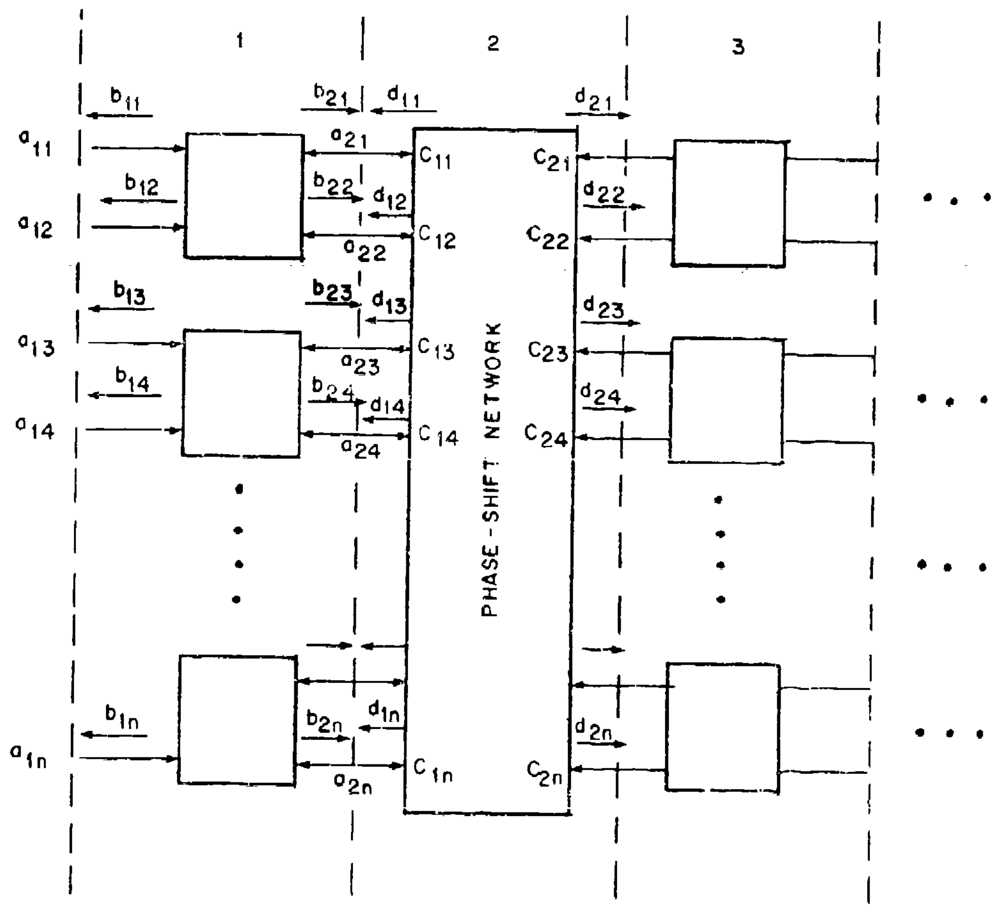


Fig. 2 — Block diagram of a Butler network

and

$$S_{12} = S_{21} = \begin{bmatrix} \alpha & -j\alpha & 0 & 0 & \dots & & \dots \\ -j\alpha & \alpha & 0 & 0 & \dots & & \dots, \\ 0 & 0 & \alpha & -j\alpha & 0 & 0 & \dots \\ 0 & 0 & -j\alpha & \alpha & 0 & 0 & \dots \\ \dots & & \dots & & \dots & & \dots \\ 0 & 0 & \dots & & \dots & \alpha & -j\alpha \\ 0 & 0 & \dots & & \dots & -j\alpha & \alpha \end{bmatrix} . \tag{8d}$$

Equation (7) can now be simplified to

$$\begin{bmatrix} b_1 \\ \hline b_2 \end{bmatrix} = \left[\begin{array}{c|c} S_{11} & S_{12} \\ \hline S_{21} & S_{22} \end{array}\right] \begin{bmatrix} a_1 \\ \hline a_2 \end{bmatrix} . \tag{9}$$

The scattering matrix in region 2, which is a phase-shift and transfer network, can be represented as

$$\begin{bmatrix} d_1 \\ \hline d_2 \end{bmatrix} = \left[\begin{array}{c|c} R_{11} & R_{12} \\ \hline R_{21} & R_{22} \end{array}\right] \begin{bmatrix} c_1 \\ \hline c_2 \end{bmatrix} , \tag{10}$$

where d_1, d_2, c_1, and c_2 are vectors such that

$$d_1 = \begin{bmatrix} d_{11} \\ d_{12} \\ \cdot \\ \cdot \\ \cdot \\ d_{1n} \end{bmatrix} , d_2 = \begin{bmatrix} d_{21} \\ d_{22} \\ \cdot \\ \cdot \\ \cdot \\ d_{2n} \end{bmatrix} , \tag{11a}$$

$$c_1 = \begin{bmatrix} c_{11} \\ c_{12} \\ \cdot \\ \cdot \\ \cdot \\ c_{1n} \end{bmatrix} , c_2 = \begin{bmatrix} c_{21} \\ c_{22} \\ \cdot \\ \cdot \\ \cdot \\ c_{2n} \end{bmatrix} . \tag{11b}$$

Matrices R_{11} and R_{22} are zero, and matrices R_{12} and R_{21} have identical elements. These matrices describe the phase shifts and interconnections from one row of couplers to the

next. Their elements depend on the configuration of the Butler network. As an example, the R matrix of the 4-port Butler network shown in Fig. 3 is

$$R_{12} = R_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & e^{-j\frac{\pi}{4}} & 0 \\ 0 & e^{-j\frac{\pi}{4}} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{12}$$

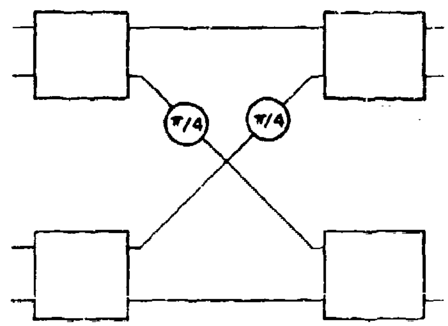


Fig. 3 — Four-port Butler network

Since we are interested in the overall scattering matrix of this network, we must first convert the scattering matrix in each region to a transfer matrix, which in turn can be multiplied to form the overall transfer matrix of the whole network. A transfer matrix can be represented as

$$\begin{bmatrix} b_2 \\ \hline a_2 \end{bmatrix} = \left[\begin{array}{c|c} T_{11} & T_{12} \\ \hline T_{21} & T_{22} \end{array}\right] \begin{bmatrix} a_1 \\ \hline b_1 \end{bmatrix}, \tag{13}$$

where a_1 and b_1 are the incident and reflected waves at the left hand ports and a_2 and b_2 are similar waves at the right hand ports.

It can be shown that a matrix T is related to an S matrix by the following relations [3,4]:

$$T_{11} = S_{21} - S_{22} S_{12}^{-1} S_{11}, \tag{14a}$$

$$T_{12} = S_{22} S_{12}^{-1}, \tag{14b}$$

$$\mathbf{T}_{21} = -\mathbf{S}_{12}^{-1}\mathbf{S}_{11}, \tag{14c}$$

and

$$\mathbf{T}_{22} = \mathbf{S}_{12}^{-1}. \tag{14d}$$

The overall transfer matrix is

$$\mathbf{T} = \prod_{i=1}^{k} \mathbf{T}_i \tag{15}$$

where $\mathbf{T}_1, \mathbf{T}_2, ..., \mathbf{T}_k$ are transfer matrices in regions 1, 2, ..., k.

The overall transfer matrix can be converted to a scattering matrix by the relations

$$\mathbf{S}_{11} = -\mathbf{T}_{22}^{-1}\mathbf{T}_{21}, \tag{16a}$$

$$\mathbf{S}_{12} = \mathbf{T}_{22}^{-1}, \tag{16b}$$

$$\mathbf{S}_{21} = \mathbf{T}_{11} - \mathbf{T}_{12}\mathbf{T}_{22}^{-1}\mathbf{T}_{21}, \tag{16c}$$

and

$$\mathbf{S}_{22} = \mathbf{T}_{12}\mathbf{T}_{22}^{-1}. \tag{16d}$$

Since $\mathbf{S}_{12} = \mathbf{S}_{21}$, one may use the simpler relation of Eq. (16b) instead of Eq. (16c).

Elements of matrix $\mathbf{S}_{21}$ (or $\mathbf{S}_{12}$) represent the transmitted waves at the output ports when a unit incident wave is applied at any one of the input ports. Therefore, matrix $\mathbf{S}_{21}$ is the transfer function of a conventional Butler network. Elements of matrix $\mathbf{S}_{11}$ (or $\mathbf{S}_{22}$) represent the reflected waves at the input ports when a unit incident wave is applied at any one of the input ports. In a reflective Butler network both the reflected waves and transmitted waves emerge from the same set of ports. Therefore, the scattering matrix of such a network is the sum of matrices $\mathbf{S}_{12}$ and $\mathbf{S}_{11}$, or

$$\mathbf{S} = \mathbf{S}_{11} + \mathbf{S}_{12}. \tag{17}$$

In deriving this relation, we have made the assumption that the symmetry plane of a reflective Butler network exhibits an open-circuit unity reflection coefficient.

PATTERNS OF AN ARRAY FED BY A BUTLER NETWORK

Figure 4 shows a schematic diagram of a reflective Butler network, which has half the components of a conventional Butler network. There are n ports, since ports a_{11},

$a_{12}, \ldots, a_{1n}$ are identical with ports $a_{21}, a_{22}, \ldots, a_{2n}$. Using previously developed notation and setting $[b_2] = [a_2] = 0$, this can be represented as

$$[b_1] = [S_{11} + S_{12}][a_1]. \tag{18}$$

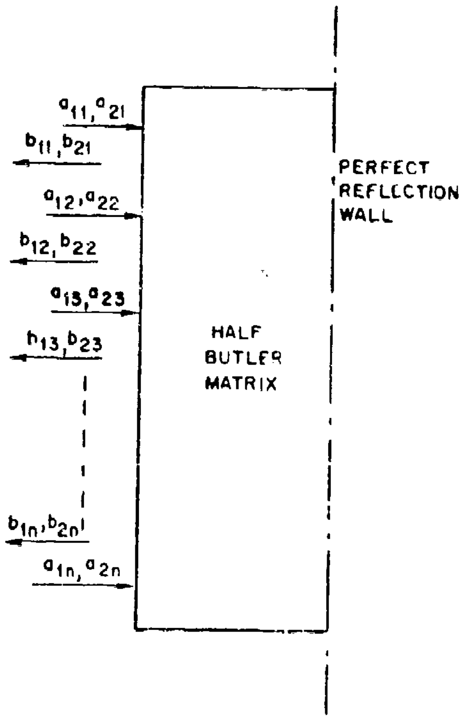


Fig. 4 — Reflective Butler network

The vector input of $[a_1]$ can be represented, for the case of an incident plane wave received by a linear array, by

$$a_{1k} = A_k \exp[j(k-1)u] \tag{19}$$

where $u = 2\pi d \sin\theta/\lambda$,

with λ = wavelength,
θ = angle of incidence from the normal to the array, and
d = element spacing.

In the subsequent discussion, we shall assume that the array has a uniform illumination function, that is, that $A_k = 1$. The scattering matrix $[S_{11}] + [S_{12}]$ is computed as a function of isolation factor I. Radiation patterns of the network-fed array are represented by two types of plot. One shows the main beams formed by several ports of the reflective Butler network, and the other shows the complete array pattern of one port of the network, in the range $0 \leqslant u \leqslant 180°$.

Figure 5 shows the array patterns of an eight-port reflective Butler network. Figure 5a shows four of the main beams for variation of the isolation factor of the 3-dB hybrid from 10 dB to 40 dB Figure 5b shows the array pattern when the main beam is at $u = 22.5°$ for the same range of isolation factor. Figure 6 shows the corresponding patterns for a 16-port reflective network. From these figures, it can be seen that the null filling level is roughly equal to the isolation factor of the 3-dB couplers. That is, for the case of 10-dB isolation, the pattern is filled to a level of about 10 dB below its peak; and for the case of 40 dB isolation, the pattern is filled to a level of about 40 dB below its peak.

Tables 1 and 2 show computed results for eight-port and 16-port reflective Butler networks, respectively. The isolation factors in dB are listed in the first column. The transmitted power is the percentage of incident power, averaged over all inputs and outputs, that would emerge from the outputs for the conventional Butler-network configuration. The remaining power emerges from the input ports. It is seen that the transmitted power decreases as the isolation decreases and as the number of rows of couplers in the network increases. For the reflective-network configuration, the input and output ports are combined, and the components emerging from these ports are also combined. The RMS amplitude and phase errors describe the effects of these spurious components on the combined outputs and are defined by

$$\Delta b = \left[\frac{\sum_{k=1}^{N} \sum_{\ell=1}^{N} \left(|s_{k\ell}| - \bar{s} \right)^2}{N^2} \right]^{1/2}$$

and

$$\Delta \phi = \left[\frac{\sum_{k=1}^{N} \sum_{\ell=1}^{N} \left(\phi_{k\ell} - \phi'_{k\ell} \right)^2}{N^2} \right]^{1/2},$$

where Δb and $\Delta\phi$ are the RMS amplitude and phase errors, respectively, $s_{k\ell}$ is an element of the scattering matrix S,

$$\bar{s} = \sum_{k=1}^{N} \sum_{\ell=1}^{N} |s_{k\ell}|/N^2,$$

$\phi_{k\ell}$ is the phase of $s_{k\ell}$, and $\phi'_{k\ell}$ is the phase of $s_{k\ell}$ for the ideal network with no errors. The error components increase with the number of rows of couplers and with decreasing isolation.

A computer program for carrying out these calculations is listed in the appendix. In addition to providing for imperfect reverse parameters of the hybrid couplers, the program provides for imperfect forward parameters and for errors in the interconnecting transmission lines.

CONCLUSIONS

An exact analysis procedure has been developed that is applicable to both conventional and reflective Butler networks with imperfect components. The analytical procedure has been programmed for computation of results for conventional and reflective Butler networks of arbitrary size. Results are presented for eight-port and 16-port reflective networks using hybrid couplers with varying degrees of isolation. The results are given in the form of radiation-pattern factors that would be obtained from a linear antenna array fed by the network and also in terms of the RMS phase and amplitude errors of the network transfer coefficients.

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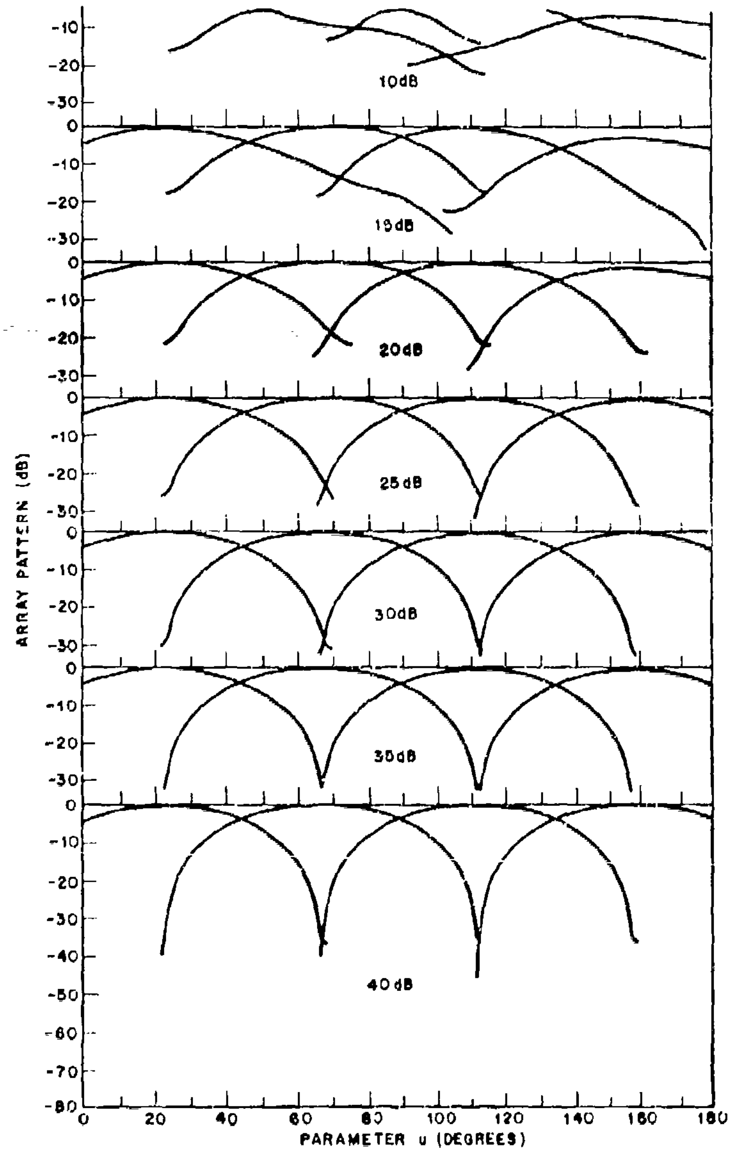

Fig. 5a — Main-beam pattern of an eight-port reflective Butler network; isolation factor varies from 10 dB to 40 dB

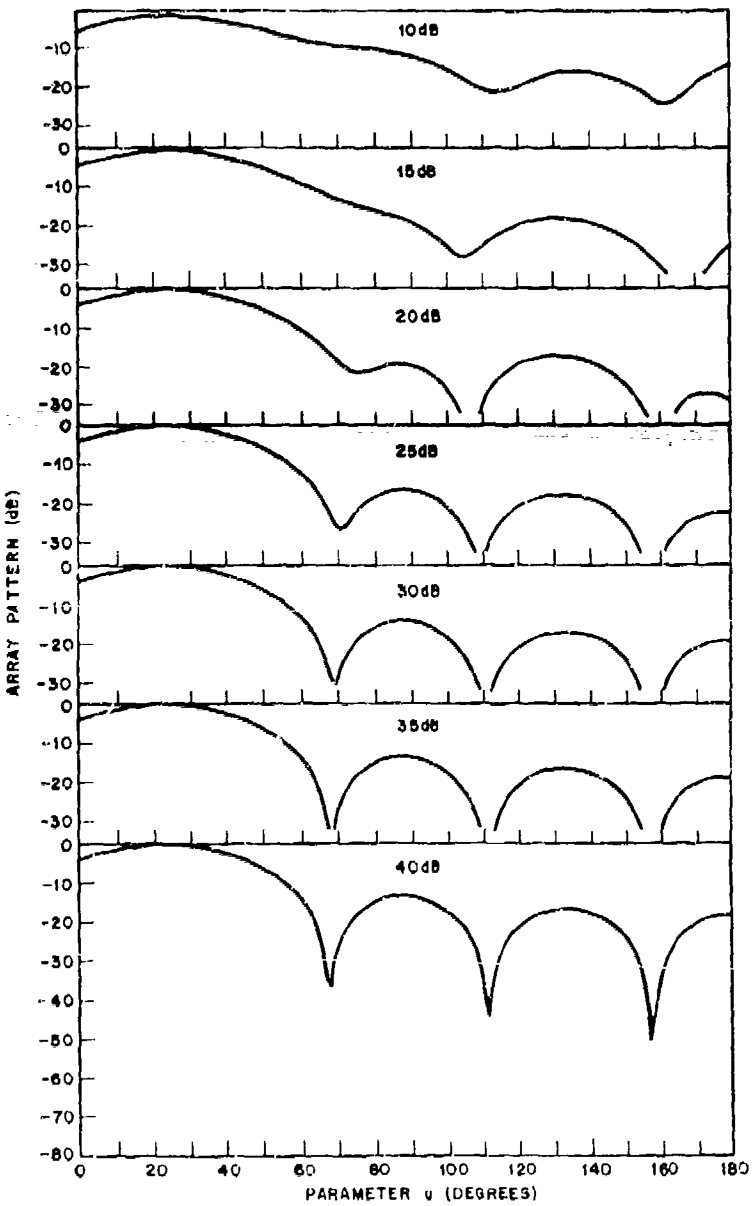

Fig. 5b — Array pattern of an eight-port reflective Butler network; isolation factor varies from 10 dB to 40 dB; main beam at $u = 22.50°$

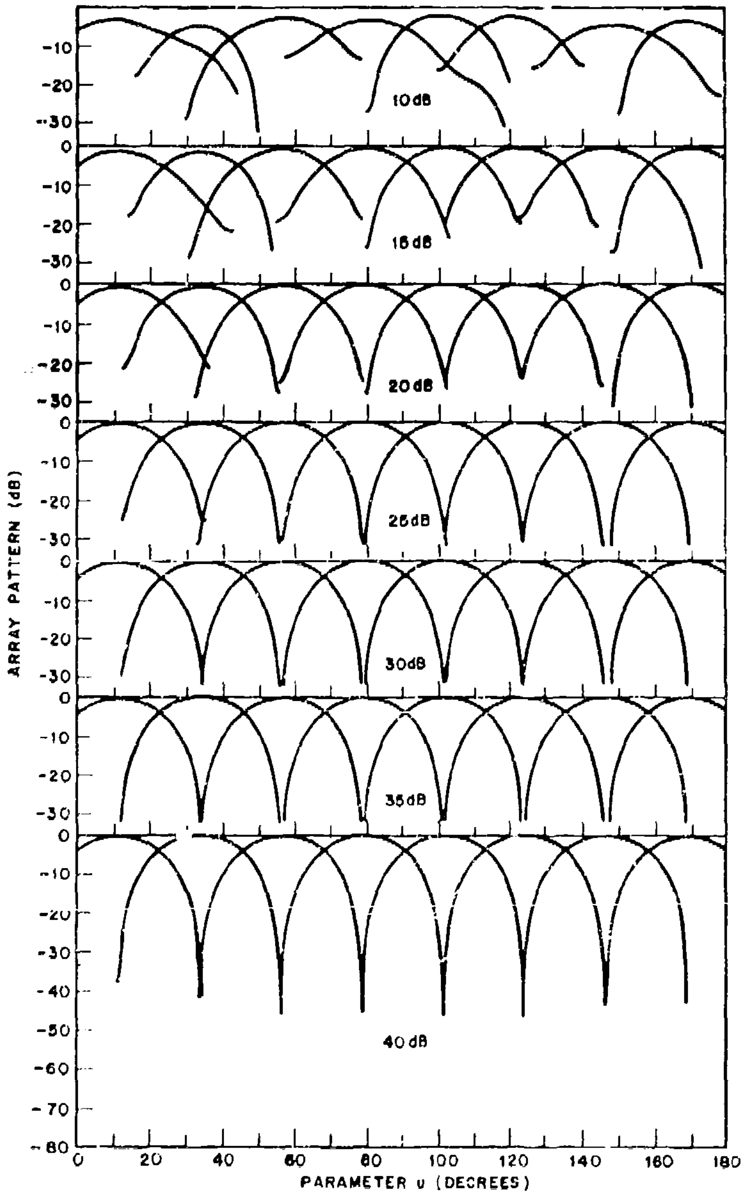

Fig. 6a — Main beam pattern of a 16-port reflective Butler network; isolation factor varies from 10 dB to 40 dB

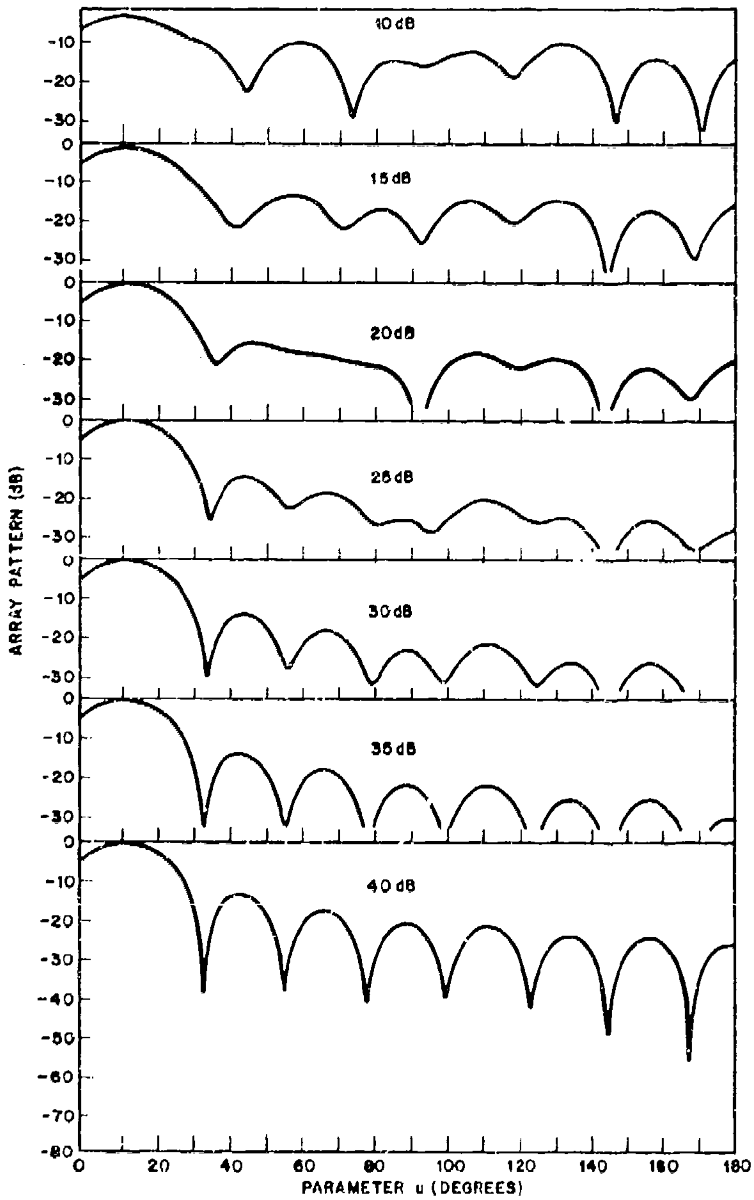

Fig. 6b — Array pattern of a 16-port reflective Butler network; isolation factor varies from 10 to 40 dB; main beam at $u = 11.25^\circ$

Table 1 — Computed Statistical Parameters for Eight-Port Reflective Network

Isolation (dB)	Transmitted Power (percent of incident)	RMS Amplitude Error (percent)	RMS Phase Error (degrees)
10	54.85	30.25	38.69
15	80.76	19.42	23.53
20	93.20	12.30	12.97
25	97.77	7.30	7.16
30	99.29	4.21	3.99
35	99.77	2.39	2.24
40	99.93	1.35	1.25

Table 2 — Computed Statistical Parameters for 16-Port Reflective Network

Isolation (dB)	Transmitted Power (percent of incident)	RMS Amplitude Error (percent)	RMS Phase Error (degrees)
10	44.92	47.29	50.58
15	76.30	31.96	30.95
20	91.49	20.41	12.66
25	97.19	11.67	6.91
30	99.10	6.61	3.84
35	99.71	3.73	2.16
40	99.91	2.10	1.22

Appendix

COMPUTER PROGRAM FOR ANALYSIS

This computer program computes the coupling coefficients from the input ports to the output ports and the power transmitted and reflected; it also plots the array radiation pattern if it is desired. The type of Butler matrix analyzed by this program can be either a conventional or a reflective type as described in this report. For this program three input data cards are required. The first data card enters the following fixed-point (I5 format) data:

NPT — Number of ports of the Butler matrix to be computed.

NROW — Number of rows of this network.

KLL — Absolute value of KLL represents the beam index whose pattern is to be plotted. If KLL = 0, there is no plot. If KLL is less than 0, the program plots the array pattern and also plots all main beams formed by the Butler matrix network.

LPRINT — Printout control. If LPRINT = 0, the program prints all detailed output at each computation step.

The second data card, which is also in a fixed-point I5 format, specifies the number of ports in each basic coupling network in each row. This implies that identical coupling networks are used in each row. However, coupling networks of different ports may be used in different rows.

The third input data card, which has a F10.6 floating-point format, specifies the coupling coefficients of the 3-dB coupler used as the basic building block of the Butler matrix network. These coefficients are read in the sequence A1, B1, C1, D1. These numbers are related to the coupling coefficient of the 3-dB coupler by the relations (see Fig. 1a)

$$\beta_1 = 10^{-(0.05 \times A1)},$$
$$\beta_2 = 10^{-(0.05 \times B1)},$$
$$\alpha_1 = 10^{-(0.05 \times C1)},$$

and

$$\alpha_2 = 10^{-(0.05 \times D1)}.$$

```
      PROGRAM RFBMTX
C     THIS PROGRAM FIRST FIGURES OUT BUTLER MATRIX CONNECTION AND PHASE
C     ANGLE , COMPUTES THE TRANSFER FUNCTION AND THEN PLOT THE PATTERN
C     MATRIX LIMIT TO THE SIZE OF 64
C     COMPILED ON JULY 13,1976 BY J. K. HSIAO
C     REVISED ON AUGUST 18,1976 BY J. K. HSIAO
C     ABSOLUTE VALUE OF KLL REPRESENTS THE BEAM INDEX WHOSE PATTERN IS
C     TO BE PLOTTED
C     KLL=0  NO PLOT
C     KLL GRATER THAN 0 PLOT PATTERN ONLY
C     KLL LESS THAN 0 PLOT BOTH PATTERN AND MAIN BEAMS
C     LLL=1, FULL MATRIX, LLL=0 REFLECTIVE MATRIX
C     LPRINT =0, PRINT ALL DETAILED OUTPUTS
C     IF LPRINT NOT EQUAL 0 NC MATRIX MULTIPLICATION RESULT IS PRINTED
C     IF LPRINT LT 0 PRINT ONLY THE TRANSFER FUNCTION
      COMMON/CS1/PLTAY(500)
      COMMON/CS4/A1,A2,B1,B2
      DIMENSION NBP(16),NBK(16)
      DIMENSION MC(8,64),PHA(8,64)
      DIMENSION S11(32,32),S12(32,32),S21(32,32),S22(32,32)
      COMPLEX S11,S12,S21,S22
      CALL PLOTS(PLTAY,500,0.)
      NMAX=32
      KC=0
    1 READ 100,NTP,NROW,KLL,LPRINT,LLL
      IF(NTP.EQ.0)GO TO 2
    3 READ 100,(NBP(I),I=1,NROW)
  100 FORMAT(16I5)
      READ 101, A1,A2,B1,B2
  101 FORMAT(8F10.6)
      IF(KC.GT.0)CALL ORIGIN(14.,0.)
      KC=KC+1
      NR1=NROW+1
      CALL NTWK(NTP,NR1,NBP,NBK,MC,PHA)
      IF(LLL.GT.0)GO TO 4
      CALL HLFMTX(NTP,NR1, NBP,NBK,MC,PHA)
    4 CALL TRFMTX(NMAX,NTP,NR1,NBP,NBK,MC,PHA,S11,S12,S21,S22)
      LL=0
      CALL PRTOUT(NTP,S21,S11,LL,NMAX,LTFP,LPRINT)
      LTFP=1
      IF(KLL.EQ.0)GO TO 1
      NPAV=1
      CALL PATERN (NTP,S21,S11,KLL,NPAV,NMAX)
      GO TO 1
    2 CALL ENDPLT
      END
```

```
      SUBROUTINE PRTOUT(NTP,TRFF,TRFB,LL,NMX,LTFP,LPRINT)
C     LLGT.0 FOR BLOCK, AND LLIS THE BLOCK NUMBER
C     LL=0 FOR OVERAL TRANSFER FUNCTION
      DIMENSION TRFF(NMX,NMX),TRFB(NMX,NMX)
      COMMON/C$4/A1,A2,B1,B2
      COMMON/C$6/AMPT(32,32),ANGL(32,32),ANGT(32),TRFF2(32,32), TR(32,32
     C),AMPAV(32),ANGAV(32),AMX(32),ANX(32),AMPRMS(32),ANGRMS(32),
     C SUMR(1824)
      COMPLEX TRFF,TRFB,TRFF2,TR,SR
      KC=0
      PI=3.1415926536
      RAC=180./PI
      K6=6
      LLL=0
      IF(A1.LE.0..OR.LL.GT.0)LLL=1
      IF(LL.LE.0)GO TO 1
      PRINT 101,LL
  101 FORMAT(//,20X,'THIS IS THE TRANSFER FUNCTION OF BLOCK',I5)
      GO TO 4
    1 PRINT 111
      PRINT 106
  106 FORMAT(//,20X,'OVERAL TRANSFER FUNCTION')
    2 IF(A1.GT.0.)GO TO 3
      PRINT 119
  119 FORMAT(//,10X,'ZERO REFLECTION')
C     GENERATE TRANSFER FUNCTION FOR AN IDEAL BUTLER MATRIX
    3 PRINT 124,NTP,A1
      PRINT 117
  124 FORMAT(/,20X,'NUMBER OF PORTS',I5,5X,'ISOLATION(DB)',F10.4,//)
      CALL TRFIDL(NTP)
      IF(LTFP.GT.0)GO TO 4
      IF(LPRINT .GT.0)GO TO 4
      PRINT 107,((AMPT(I,J),J=1,NTP),I=1,NTP)
      PRINT 117
      PRINT 107,((ANGL(I,J),J=1,NTP),I=1,NTP)
      PRINT 117
    4 IF(A1.LE.0.)K6=2
      DO 60 K=1,K6
      SUM=0.
      DO 15 I=1,NTP
   15 SUMR(I)=0.
      IF(LPRINT .GT.0)GO TO 75
      GO TO (71,72,73,74,76,77)K
   71 PRINT 102
  102 FORMAT(//,20X,'AMPLITUDE OF FORWARD TRANSFER FUNCTION')
      PRINT 117
  117 FORMAT(/)
      GO TO 75
   72 PRINT 103
  103 FORMAT(//,20X,'PHASE ANGLE OF FORWARD TRANSFER FUNCTION')
```

```
      PRINT 117
      GO TO 75
   73 PRINT 104
  104 FORMAT(//,20X,'AMPLITUDE OF REFLECTIVE TRANSFER FUNCTION')
      PRINT 117
      GO TO 75
   74 PRINT 105
  105 FORMAT(//,20X,'PHASE ANGLE OF REFLECTIVE TRANSFER FUNCTION')
      PRINT 117
      GO TO 75
   76 PRINT 109
  109 FORMAT(//,20X,'AMPLITUDE OF THE RESULTANT TRANSFER FUNCTION')
      PRINT 117
      GO TO 75
   77 PRINT 110
  110 FORMAT(//,20X,'PHASE ANGLE OF THE RESULTANT TRANSFER FUNCTION')
      PRINT 117
   75 DO 67 I=1,NTP
      DO 70 J=1,NTP
      GO TO(61,62,63,64,65,66)K
   61 ANGT(J)=CABS(TRFF(I,J))
      ANGT2=ANGT(J)**2
      SUM=SUM+ANGT2
      SUMR(J)=SUMR(J)+ANGT2
      GO TO 70
   62 IF(LPRINT .GT.0)GO TO 70
      ANGT(J)=CANG(TRFF(I,J))*RAC
      GO TO 70
   63 ANGT(J)=CABS(TRFB(I,J))
      ANGT2=ANGT(J)**2
      SUM=SUM+ANGT2
      SUMR(J)=SUMR(J)+ANGT2
      GO TO 70
   64 IF(LPRINT .GT.0)GO TO 70
      ANGT(J)=CANG(TRFB(I,J))*RAC
      GO TO 70
   65 TR(I,J)    =TRFF(I,J)+TRFB(I,J)
      ANGT(J)=CABS(TR(I,J))
      ANGT2=ANGT(J)**2
      SUMR(J)=SUMR(J)+ANGT2
      SUM=SUM+ANGT2
      IF(LLL.GT.0)GO TO 70
      AMPT(I,J)=(ANGT(J)-AMPT(I,J))/AMPT(1,J)
      GO TO 70
   66 ANGT(J)=CANG(TR(I,J))*RAC
      IF(LLL.GT.0)GO TO 70
      AG        =ANGT(J)-ANGL(I,J)
      ANGL(J,1)=AG
      IF(ABS(AG).LE.180.)GO TO 70
      NSIGN=1
```

```
      IF(AG.GT.0.)NSIGN=-1
      ANGL(I,J)=NSIGN*(360.-ABS(AG))
70    CONTINUE
      IF(LPRINT .GT.0)GO TO 67
      PRINT 107,(ANGT(J),J=1,NTP)
107   FORMAT(10X, 8F10.4)
67    CONTINUE
      KMOD=MOD(K,2)
      IF(KMOD.LE.0)GO TO 60
      PRINT 122,SUM
122   FORMAT(//,10X,'TOTAL POWER OUTPUT',F10.4)
      PRINT 123,(SUMR(I),I=1,NTP)
123   FORMAT(//,10X,'POWER FROM EACH PORT',/,(10X,10F10.4))
60    CONTINUE
      IF(LLL.GT.0)GO TO 7
      IF(LPRINT .GT.0)GO TO 8
      DO 50 L=1,2
      GO TO (51,52)L
51    PRINT 120
120   FORMAT(//,20X,'ERROR FUNCTION',//,20X,'AMPLITUDE',/)
      GO TO 53
52    PRINT 121
121   FORMAT(//,20X,'PHASE ANGLE',/)
53    DO 50 I=1,NTP
      GO TO (54,55)L
54    PRINT 107, (AMPT(J,I),J=1,NTP)
      GO TO 50
55    PRINT 107, (ANGL(J,I),J=1,NTP)
50    CONTINUE
      IF(LPRINT .LT.0)RETURN
      PRINT 111
8     DO 59 L=1,2
      DO 57 I=1,NTP
      IF(L.GT.1.AND.I.GT.1)GO TO 58
      ANGS=0.
      AMPS=0.
      ANGX=0.
      AMPX=0.
58    DO 56 J=1,NTP
      AMPS=AMPS+AMPT(J,I)
      ANGS=ANGS+ANGL(J,I)
      IF(AMPT(J,I).GT.AMPX)AMPX=AMPT(J,I)
56    IF(ABS(ANGL(J,I)).GT.ABS(ANGX))ANGX=ANGL(J,I)
      IF(L.GT.1)GO TO 57
      AMPAV(I)=AMPS/NTP
      ANGAV(I)=ANGS/NTP
      AMX(I)=AMPX
      ANX(I)=ANGX
57    CONTINUE
      IF(L.GT.1)GO TO 59
```

```
      PRINT 117
      PRINT 107,(AMPAV(K),K=1,NTP)
      PRINT 117
      PRINT 107,(ANGAV(K),K=1,NTP)
      PRINT 117
      PRINT 107,(AMX(K),K=1,NTP)
      PRINT 117
      PRINT 107,(ANX(K),K=1,NTP)
      PRINT 117
59    CONTINUE
      AMPS=AMPS/NTP**2
      ANGS=ANGS/NTP**2
      ANGSST=0.
      AMPSST=0.
      DO 80 I=1,NTP
      ANGSS=0.
      AMPSS=0.
      DO 81 J=1,NTP
      AMPSS=AMPSS+(AMPT(J,I)-AMPAV(I))**2
      ANGSS=ANGSS+(ANGL(J,I)-ANGAV(I))**2
      AMPSST=AMPSST+(AMPT(J,I)-AMPS)**2
81    ANGSST=ANGSST+(ANGL(J,I)-ANGS)**2
      AMPRMS(I)=SQRT(AMPSS /NTP)
      ANGRMS(I)=SQRT(ANGSS /NTP)
80    CONTINUE
      PRINT 107,(AMPRMS(K),K=1,NTP)
      PRINT 117
      PRINT 107,(ANGRMS(K),K=1,NTP)
      AMPSS=SQRT(AMPSST/NTP**2)
      ANGSS=SQRT(ANGSST/NTP**2)
      PRINT 117
      PRINT 107,AMPS,ANGS,AMPX,ANGX,AMPSS,ANGSS
7     IF(LL.GT.0)RETURN
      IF(LPRINT .NE.0)RETURN
      PRINT 111
111   FORMAT(1H1)
      L3=K6/2
6     DO 10 L=1,L3
      GO TO (11,12,13)L
11    PRINT 112
112   FORMAT(//,20X,'IDEAL CASE',//)
      GO TO 14
12    PRINT 113
113   FORMAT(//,20X,'ACTUAL CASE',//)
      GO TO 14
13    PRINT 114
114   FORMAT(//,20X,'DIFFERENCE',//)
14    DO 30 I=1,NTP
      DO 30 J=1,NTP
      SR=CMPLX(0.,0.)
```

```
      DO 40 K=1,NTP
      GO TO (41,42,43)L
41    SR=SR+TRFF(I,K)*CONJG(TRFF2(J,K))
      GO TO 40
42    SR=SR+TR(I,K)*CONJG(TRFF2(J,K))
      GO TO 40
43    SR=SR+TRFB(I,K)*CONJG(TRFF2(J,K))
40    CONTINUE
      ANGL(I,J)=CANG(SR)*RAC
30    AMPT(I,J)=CABS(SR)
      DO 20 K=1,2
      GO TO (21,22)K
21    PRINT 115
115   FORMAT(20X,'AMPLITUDE',/)
      GO TO 23
22    PRINT 116
116   FORMAT(//,20X,'PHASE ANGLE',/)
23    DO 20 I=1,NTP
      GO TO (24,25)K
24    PRINT 107, (AMPT(I,J),J=1,NTP)
      GO TO 20
25    PRINT 107, (ANGL(I,J),J=1,NTP)
20    CONTINUE
10    CONTINUE
      IF( KC.GT.0)RETURN
      IF(A1.LE.0.)RETURN
      PRINT 118
118   FORMAT(1H1,10X,'REFLECTION MATRIX IS USED',//)
      DO 5 I=1,NTP
      DO 5 J=1,NTP
5     TRFF2(I,J)=TRFF(I,J)
      KC=KC+1
      GO TO 6
      END
```

```
      SUBROUTINE TRFMTX(NM,NN,NR1,NBP,NBK,MC,PHA,S11,S12,S21,S22)
      DIMENSION NBP(16),NBK(16)
      DIMENSION MC(NR1,NN),PHA(NR1,NN)
      DIMENSION S11(NM,NM),S12(NM,NM),S21(NM,NM),S22(NM,NM)
      COMMON/CS5/T11(32,32),T12(32,32),T21(32,32),T22(32,32)
      COMMON/CS6/R11(32,32),R12(32,32),R21(32,32),R22(32,32)
      DIMENSION SS11(8,8),SS12(8,8),SS21(8,8),SS22(8,8)
      DIMENSION MCT(32)
      COMPLEX S11,S12,S21,S22,T11,T12,T21,T22,R11,R12,R21,R22,AR,SS11,
     CSS12,SS21,SS22
C     FIRST INDEX ROW
C     SECOND INDEX COLUMN
      DO 10 I=1,NR1
C     TRANSFER MATRIX IN CONNECTION REGION
      DO 11 L=1,NN
      LL=MC(I,L)
   11 MCT(LL)=L
      PRINT 102,(MC(I,L),L=1,NN)
      PRINT 102,(MCT(L),  L=1,NN)
      PRINT 101,(PHA(I,L),L=1,NN)
  102 FORMAT(//,(10X,8I5))
  101 FORMAT(//,(10X,8F10.4))
      DO 20 J=1,NN
      DO 20 K=1,NN
      T11(J,K)=CMPLX(0.,0.)
      T12(J,K)=CMPLX(0.,0.)
      T21(J,K)=CMPLX(0.,0.)
      T22(J,K)=CMPLX(0.,0.)
      IF(MCT(J).NE.K)GO TO 20
      T11(J,K)=AR(PHA(I,J))
      T22(J,K)=CONJG(T11(J,K))
   20 CONTINUE
      PRINT 100,((T11(M,N),N=1,NN),M=1,NN)
      PRINT 100,((T22(M,N),N=1,NN),M=1,NN)
      IF(I.GT.1)GO TO 21
      DO 22 J=1,NN
      DO 22 K=1,NN
      R11(J,K)=T11(J,K)
      R12(J,K)=CMPLX(0.,0.)
      R21(J,K)=CMPLX(0.,0.)
   22 R22(J,K)=T22(J,K)
      GO TO 23
   21 CALL MTXMLT(NM,NN,    T11,T12,T21,T22,R11,R12,R21,R22)
   23 PRINT 100,((R11(M,N),N=1,NN),M=1,NN)
      PRINT 100,((R12(M,N),N=1,NN),M=1,NN)
      PRINT 100,((R21(M,N),N=1,NN),M=1,NN)
      PRINT 100,((R22(M,N),N=1,NN),M=1,NN)
  100 FORMAT(//,(10X,8F10.4))
      IF(I.EQ.NR1)GO TO 10
C     TRANFER MATRIX IN BLOCK REGION
```

```
      NP=NBP(I)
      IF(I.LE.1)GO TO 26
      IF(NP.EQ.NBP(I-1))GO TO 27
 26   CALL BLK(8,NP,SS11,SS12,SS21,SS22)
C     RESET S MATRIX
      DO 24 J=1,NN
      DO 24 K=1,NN
 24   S11(J,K)=CMPLX(0.,0.)
      S12(J,K)=CMPLX(0.,0.)
      S21(J,K)=CMPLX(0.,0.)
      S22(J,K)=CMPLX(0.,0.)
      DO 25 J=1,NN,NP
      DO 25 JJ=1,NP
      J1=JJ-1
      DO 25 KK=1,NP
      K1=KK-1
      S11(J+J1,J+K1)=SS11(JJ,KK)
      S12(J+J1,J+K1)=SS12(JJ,KK)
      S21(J+J1,J+K1)=SS21(JJ,KK)
      S22(J+J1,J+K1)=SS22(JJ,KK)
 25   CONTINUE
      PRINT 100,((S11(M,N),N=1,NN),M=1,NN)
      PRINT 100,((S12(M,N),N=1,NN),M=1,NN)
      PRINT 100,((S21(M,N),N=1,NN),M=1,NN)
      PRINT 100,((S22(M,N),N=1,NN),M=1,NN)
C     INVERSE S-MATRIX
      CALL INVS1(NM,NN,S12)
      CALL STTRF(NM,NN,S11,S12,S21,S22,T11,T12,T21,T22)
      PRINT 100,((S11(M,N),N=1,NN),M=1,NN)
      PRINT 100,((S12(M,N),N=1,NN),M=1,NN)
      PRINT 100,((S21(M,N),N=1,NN),M=1,NN)
      PRINT 100,((S22(M,N),N=1,NN),M=1,NN)
 27   DO 50 J=1,NN
      DO 50 K=1,NN
      T11(J,K)=S11(J,K)
      T12(J,K)=S12(J,K)
      T21(J,K)=S21(J,K)
 50   T22(J,K)=S22(J,K)
      CALL MTXMLT(NM,NN,T11,T12,T21,T22,R11,R12,R21,R22)
      PRINT 100,((R11(M,N),N=1,NN),M=1,NN)
      PRINT 100,((R12(M,N),N=1,NN),M=1,NN)
      PRINT 100,((R21(M,N),N=1,NN),M=1,NN)
      PRINT 100,((R22(M,N),N=1,NN),M=1,NN)
 10   CONTINUE
      CALL INVS1(NM,NN,R22)
      DO 40 J=1,NN
      DO 40 K=1,NN
      S12(J,K)=R22(J,K)
      S21(J,K)=R22(J,K)
      S11(J,K)=CMPLX(0.,0.)
      S22(J,K)=CMPLX(0.,0.)
      DO 40 L=1,NN
      S11(J,K)=S11(J,K)-R22(J,L)*R21(L,K)
      S22(J,K)=S22(J,K)+R12(J,L)*R22(L,K)
 40   CONTINUE
      PRINT 100,((S11(M,N),N=1,NN),M=1,NN)
      PRINT 100,((S12(M,N),N=1,NN),M=1,NN)
      PRINT 100,((S21(M,N),N=1,NN),M=1,NN)
      PRINT 100,((S22(M,N),N=1,NN),M=1,NN)
      RETURN
      END
```

```
0001          SUBROUTINE STRF(NM,NN,NR1,NBP,NBK,MC,PHA,S11,S12,S21,S22)
0002          DIMENSION NBP(16),NBK(16)
0003          DIMENSION MC(NR1,NN),PHA(NR1,NN)
0004          DIMENSION S11(NM,NM),S12(NM,NM),S21(NM,NM),S22(NM,NM)
0005          COMMON/C$5/T11(8,8),T12(8,8),T21(8,8),T22(8,8),R11(8,8),R12(8,8),
             C R21(8,8),R22(8,8),SPACE(7168)
0006          DIMENSION MCT(32)
0007          DIMENSION SS11(2,2),SS12(2,2),SS21(2,2),SS22(2,2)
0008          COMPLEX S11,S12,S21,S22,T11,T12,T21,T22,R11,R12,R21,R22,AR,SS11,
             CSS12,SS21,SS22
0009          CALL TWOPT(SS11,SS12,SS21,SS22,2)
        C     1ST INDEX,COLUMN
        C     2ND INDEX,ROW
0010          DO 10 I=1,NR1
        C     TRANSFER MATRIX IN CONNECTION REGION
0011          DO 11 L=1,NN
0012          LL=MC(I,L)
0013     11   MCT(LL)=L
0014          DO 20 J=1,NN
0015          DO 20 K=1,NN
0016          T11(J,K)=CMPLX(0.,0.)
0017          T12(J,K)=CMPLX(0.,0.)
0018          T21(J,K)=CMPLX(0.,0.)
0019          T22(J,K)=CMPLX(0.,0.)
0020          IF(MCT(J).NE.K)GO TO 20
0021          T11(J,K)=AR(PHA(I,J))
0022          T22(J,K)=CONJG(T11(J,K))
0023     20   CONTINUE
0024          IF(I.GT.1)GO TO 21
0025          DO 22 J=1,NN
0026          DO 22 K=1,NN
0027          R11(J,K)=T11(J,K)
0028          R12(J,K)=T12(J,K)
0029          R21(J,K)=T21(J,K)
0030     22   R22(J,K)=T22(J,K)
0031          GO TO 23
0032     21   CALL MTXMLT(NM,NN,    T11,T12,T21,T22,R11,R12,R21,R22)
0033          IF(I.EQ.NR1)GO TO 10
        C     TRANFER MATRIX IN BLOCK REGION
        C     RESET S MATRIX
0034     23   NP=NBP(I)
0035          IF(I.LE.1)GO TO 26
0036          IF(NP.EQ.NBP(I-1))GO TO 27
0037     26   DO 24 J=1,NN
0038          DO 24 K=1,NN
0039     24   S11(J,K)=CMPLX(0.,0.)
0040          S12(J,K)=CMPLX(0.,0.)
0041          S21(J,K)=CMPLX(0.,0.)
0042          S22(J,K)=CMPLX(0.,0.)
0043          DO 25 J=1,NN,NP
```

```
      DO 25 JJ=1,NP
      J1=JJ-1
      DO 25 KK=1,NP
      K1=KK-1
      S11(J+J1,J+K1)=SS11(JJ,KK)
      S12(J+J1,J+K1)=SS12(JJ,KK)
      S21(J+J1,J+K1)=SS21(JJ,KK)
      S22(J+J1,J+K1)=SS22(JJ,KK)
   25 CONTINUE
C     INVERSE S-MATRIX
      CALL INVS2(NM,NN,S12)
      CALL STTRF(NM,NN,S11,S12,S21,S22,T11,T12,T21,T22)
   27 DO 50 J=1,NN
      DO 50 L=1,NN
      T11(J,K)=S11(J,K)
      T12(J,K)=S12(J,K)
      T21(J,K)=S21(J,K)
   50 T22(J,K)=S22(J,K)
      CALL MTXMLT(NM,NN,T11,T12,T21,T22,R11,R12,R21,R22)
   10 CONTINUE
      CALL INVS2(NM,NN,R22)
      DO 40 J=1,NN
      DO 40 K=1,NN
      S12(J,K)=R22(J,K)
      S21(J,K)=R22(J,K)
      S11(J,K)=CMPLX(0.,0.)
      S22(J,K)=CMPLX(0.,0.)
      DO 40 L=1,NN
      S11(J,K)=S11(J,K)-R22(J,L)*R21(L,K)
      S22(J,K)=S22(J,K)+R12(J,L)*R22(L,K)
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STTRF(NM,NN,S11,S12,S21,S22,T11,T12,T21,T22)
C     THIS SUBROUTINE INVERSES S MATRIX AND STORES IN T
C
      DIMENSION S11(NM,NM),S12(NM,NM),S21(NM,NM),S22(NM,NM)
      DIMENSION T11(NM,NM),T12(NM,NM),T21(NM,NM),T22(NM,NM)
      COMPLEX S11,S12,S21,S22,T11,T12,T21,T22
      DO 30 J=1,NN
      DO 30 K=1,NN
      T22(J,K)=S12(J,K)
      T21(J,K)=CMPLX(0.,0.)
      DO 30 L=1,NN
   30 T21(J,K)=T21(J,K)-S12(J,L)*S11(L,K)
      DO 31 J=1,NN
      DO 31 K=1,NN
      T12(J,K)=CMPLX(0.,0.)
      T11(J,K)=S21(J,K)
      DO 31 L=1,NN
      T12(J,K)=T12(J,K)+S22(J,L)*S12(L,K)
   31 T11(J,K)=T11(J,K) +S22(J,L)*T21(L,K)
      DO 20 K=1,NN
      DO 20 J=1,NN
      S11(J,K)=T11(J,K)
      S12(J,K)=T12(J,K)
      S21(J,K)=T21(J,K)
   20 S22(J,K)=T22(J,K)
      RETURN
      END
```

```
      SUBROUTINE MTXMLT(NM,NN,R11,R12,R21,R22,T11,T12,T21,T22)
C     THIS SUBROUTINE MULTIPLE  SUBMATRICES R*T THEN STORE THE RESULT
C     IN R
C     S=R*T
C     S11=R11*T11+R12*T21
C     S12=R11*T12+R12*T22
C     S21=R21*T11+R22*T21
C     S22=R21*T12+R22*T22
      DIMENSION TT1(32,32),TT2(32,32)
      DIMENSION T11(NM,NM),T12(NM,NM),T21(NM,NM),T22(NM,NM)
      DIMENSION R11(NM,NM),R12(NM,NM),R21(NM,NM),R22(NM,NM)
      COMPLEX T11,T12,T21,T22,R11,R12,R21,R22,TT1,TT2
      PRINT 101
      PRINT 100,((R11(M,N),N=1,NN),M=1,NN)
      PRINT 100,((R12(M,N),N=1,NN),M=1,NN)
      PRINT 100,((R21(M,N),N=1,NN),M=1,NN)
      PRINT 100,((R22(M,N),N=1,NN),M=1,NN)
      PRINT 100,((T11(M,N),N=1,NN),M=1,NN)
      PRINT 100,((T12(M,N),N=1,NN),M=1,NN)
      PRINT 100,((T21(M,N),N=1,NN),M=1,NN)
      PRINT 100,((T22(M,N),N=1,NN),M=1,NN)
      PRINT 101
  100 FORMAT(//,(10X,8F10.4))
  101 FORMAT(//,'.......................................',//)
      DO 10 J=1,NN
      DO 10 K=1,NN
      TT1(J,K)=CMPLX(0.,0.)
      TT2(J,K)=CMPLX(0.,0.)
      DO 10 L=1,NN
      TT1(J,K)=TT1(J,K)+R11(J,L)*T11(L,K)+R12(J,L)*T21(L,K)
      TT2(J,K)=TT2(J,K)+R11(J,L)*T12(L,K)+R12(J,L)*T22(L,K)
   10 CONTINUE
      DO 20 J=1,NN
      DO 20 K=1,NN
      R11(J,K)=TT1(J,K)
   20 R12(J,K)=TT2(J,K)
      DO 30 J=1,NN
      DO 30 K=1,NN
      TT1(J,K)=CMPLX(0.,0.)
      TT2(J,K)=CMPLX(0.,0.)
      DO 30 L=1,NN
      TT1(J,K)=TT1(J,K)+R21(J,L)*T11(L,K)+R22(J,L)*T21(L,K)
      TT2(J,K)=TT2(J,K)+R21(J,L)*T12(L,K)+R22(J,L)*T22(L,K)
   30 CONTINUE
      DO 40 J=1,NN
      DO 40 K=1,NN
      R21(J,K)=TT1(J,K)
   40 R22(J,K)=TT2(J,K)
      DO 50 J=1,NN
      DO 50 K=1,NN
      T11(J,K)=R11(J,K)
      T12(J,K)=R12(J,K)
      T21(J,K)=R21(J,K)
   50 T22(J,K)=R22(J,K)
      RETURN
      END
```

```
      SUBROUTINE PATERN (NTP,TRFF,TRFB,KLL,NPAV,NMX)
C     ABSOLUTE VALUE OF KLL REPRESENTS THE BEAM INDEX WHOSE PATTERN IS
C     TO BE PLOTTED
C     KLL=0  NO PLOT
C     KLL GRATER THAN 0 PLOT PATTERN ONLY
C     KLL LESS THAN 0 PLOT BOTH PATTERN AND MAIN BEAMS
      COMMON/C$1/PLTAY(500)
      DIMENSION TRFF(NMX,NMX),TRFB(NMX,NMX)
      COMMON/C$5/PEAK(64,100),PMAX(64),PAV(64),PMAV(64),KIND1(64),
     C  KIND2(64),PEAKDB(100),SPACE(1372)
      COMMON/C$6/CONTA(4096),SINTA(4096)
      COMPLEX TRFF,TRFB,S
      Z(X)=10.*ALOG10(X)
      PRINT 104
  104 FORMAT(1H1)
      PI=3.1415926536
      ATR=PI/180.
      KPLOT=IABS(KLL)
      NTP2=NTP/2
C     PLOT FRAME
      YSL=80.
      NY=YSL
      XSL=180.
      NX=XSL
      HN=5.
      SY=2.
      XM=10.
      YM=5.
      YS=2.
      YSM=YS+YM
      NTA=20*NTP
      NTA1=NTA+1
      TAINC=PI/NTA
      PNOR=NTP
      CALL PHASAN(TAINC,IKA)
      KL=1
      IF(KLL.LT.0)KL=2
      NTA1N=2*NTA+1
      DO 1 I=1,NTP
      DO 1 J=1,NTP
    1 TRFF(I,J)=TRFF(I,J)+TRFB(I,J)
      DO 20 IL=1,KL
      IF(IL.LT.2)GO TO 25
      CALL PLOT(XM+4.,0.,-3)
      NTA1N=NTA1
      KSIGN=1
   25 CALL FRAME(XM,YM,XSL,YSL, SY,HN,NX,NY)
      DO 20 K=1,NTP2
      KK=0
      KFLAG=0
```

```
      KMI=1
      KPCONT=1
      KMIND=1
      KMA=0
      LEDGE=0
      DO 30 IJ=1,NTAIN
      IF(IL.GT.1)GO TO 23
      NSIGN=-1
      IF(IJ.GE.NTA1)NSIGN=1
      I=IJ-NTA1+NSIGN
      GO TO 24
 23   I=IJ
 24   II=IABS(I)
      I1=II-1
      PAR=0.
      PAI=0.
      IF(IL.LT.2)GO TO 21
      IF(I.LT.KIND1(K).OR.I.GT.KIND2(K))GO TO 30
      IF(I.EQ.KIND1(K))II=1
 21   DO 40 J=1,NTP
      S=TRFF(J,K)
      JI=(J-1)*I1+1
      JMOD=MOD(JI,IKA)
      IF(JMOD.EQ.0)JMOD=IKA
      PAR=CONTA(JMOD)*REAL(S)-SINTA(JMOD)*AIMAG(S)*NSIGN+PAR
      PAI=CONTA(JMOD)*AIMAG(S)+SINTA(JMOD)*REAL(S)*NSIGN+PAI
 40   CONTINUE
      PAT=PAR**2+PAI**2
      PAT=PAT/PNCR
      IF(IL.EQ.2)GO TO 22
      IF(IJ.LE.1)GO TO 31
      IF(PAT-PAT1)32,31,33
C     EXAMINE IF A MAXIMUM IS PASSED
 32   KMI=1
      IF(IJ.EQ.2)LEDGE=1
      IF(KMA.LE.0)GO TO 31
      IF(PAT1.LE.PEAK(K,KPCONT )) GO TO 34
      KIND1(K)=KMIND
      KFLAG=1
      KPCONT =KK+1
 34   KK=KK+1
      PEAK(K,KK)=PAT1
      KMA=0
      GO TO 31
C     EXAMINE IF A MINIMUM IS PASSED
 33   KMA=1
      IF(KMI.LE.0)GO TO 31
 35   KMIND=I-1
      IF(KFLAG.GT.0)KIND2(K)=I-1
      KFLAG=0
```

```
      KMI=0
   31 PAT1=PAT
C     PLOT PATTERN FOR A GIVEN BEAM
      IF(K.NE.KPLOT)GO TO 30
      IF(IJ.LT.NTA1)GO TO 30
   22 DB=Z(PAT)
      Y=(1.+DB/YSL)*YM+SY
      IF(Y.GT.YSM)Y=YSM
      IF(Y.LT.SY)Y=SY
      P=I-1
      X=P*XM/NTA
      IF(II.EQ.1)GO TO 3
      CALL PLOT(X,Y,2)
      GO TO 30
    3 CALL PLOT(X,Y,3)
   30 CONTINUE
      IF(IL.GE.2)GO TO 20
      IF(KMA.GT.0)GO TO 42
      IF(KPCONT .EQ.KK)KIND2(K)=NTA1
      IF(KIND2(K).LE.0)KIND2(K)=NTA1
C     DELETE THE MAIN LOBE
      GO TO 43
   42 IF(LEDGE.LE.0)GO TO 43
      KK=KK+1
      PEAK(K,KK)=PAT1
   43 DO 44 I=1,KK
   44 PEAKDB(I)=10.*ALOG10(PEAK(K,I))
      PRINT 102,K
  102 FORMAT(//,20X,'BEAM INDEX',I5)
      PRINT 101,(PEAKDB(I),I=1,KK)
      KK=KK-1
      DO 53 L=1,KK
      IF(L.LT.KPCONT )GO TO 53
      PEAK(K,L)=PEAK(K,L+1)
   53 CONTINUE
      PMAX(K)=0
      PSUM=0.
      PRINT 101,(PEAK(K,I),I=1,KK)
  101 FORMAT(//,(10X,10E12.4))
      DO 50 L=1,KK
      IF(PEAK(K,L).GT.PMAX(K))PMAX(K)=PEAK(K,L)
      PSUM=PSUM+PEAK(K,L)
   50 CONTINUE
      PAV(K)=PSUM/KK
      PMAX(K)=Z(PMAX(K))
      PAV(K)=Z(PAV(K))
      PRINT 103,PMAX(K),PAV(K)
  103 FORMAT(//,10X,'PEAK',F10.4,5X,'AVERAGE',F10.4)
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE HLFMIX(NTP,NR1,NBP,NBK,MC,PHA)
      DIMENSION NBP(16),NBK(16)
      DIMENSION MC(NR1,NTP),PHA(NR1,NTP)
      COMMON/C$3/MCT(64)
      DIMENSION ANG(64) ,ATEMP(64)
      NN=NR1/2
      LL=(NR1+1)/2-NN
C     LL=1 NUMBER OF ROWS IS EVEN
C     LL=0 NUMBER OF ROWS IS ODD
      CALL PHASUM(NR1,NTP,NBP,MC,PHA,ANG)
      DO 10 I=1,NTP
      II=I
      DO 11 J=1,NR1
      KK=MC(J,II)
      IF(J.EQ.NN)KKP=KK
      II=KK
   11 CONTINUE
C     FIND THE JOINT POINT THEN STORE IN MCT ARRAY
      DO 12 J=1,NN
      KKS=MC(J,KK)
   12 KK=KKS
      MCT(KKP)=KKS
C     AVERAGE THE PHASE ANGLES FOR SYMMETRICAL MATRIX
      DO 13 J=1,NN
      JJ=NR1-J+1
      IMC=MC(J,I)
      AVG=(PHA(J,IMC)+PHA(JJ,I))/2.
   13 PHA(J,IMC)=AVG
      PHA(JJ,I)=AVG
   10 CONTINUE
C     CORRECT PHASE ANGLE OF THE MIDDLE ROW WHEN THE NUMBER OF ROWS IS
C     EVEN
      IF(LL.LE.0)GO TO 1
      N1=NN+1
      DO 20 I=1,NTP
      II=MC(N1,I)
      IN=MC(N1,MCT(I))
      ATEMP(II)=PHA(N1,II)
      IF(PHA(N1,IN).GT.ATEMP(II))ATEMP(II)=PHA(N1,IN)
   20 CONTINUE
C     CORRECT THE PHASE ANGLE BY ADDING THE SAME EXTRA PHASE TO EACH
C     PORT IN A BLOCK
      NMP=NBP(N1)
      NMB=NBK(N1)
      DO 21 I=1,NMB
      IMB=(I-1)*NMP
      AA=0.
      DO 22 J=1,NMP
      KK=IMB+J
      A=ATEMP(KK)-PHA(N1,KK)
      IF(A.GT.AA)AA=A
   22 CONTINUE
      IF(AA.LE.0.) GO TO 21
      DO 23 J=1,NMP
      KK=IMB+J
   23 PHA(N1,KK)=PHA(N1,KK)+AA
   21 CONTINUE
      RETURN
C     CORRECT THE PHASE ANGLES FOR THE CASE WHEN THE NUMBER OF ROWS IS
C     ODD
```

```
    1  CALL PHASUM(NR1,NTP,NBP,MC,PHA,ATEMP)
       DO 30 I=1,NTP
       AA=ANG(I)-ATEMP(I)
       JJ=MC (1,I)
   30  PHA(1,JJ)=AA
       PHA(NR1,I)=AA
       RETURN
       END
```

```
       SUBROUTINE PHASUM(NR1,NTP,NBP,MC,PHA,AS)
       DIMENSION NBP(16)
       DIMENSION MC(NR1,NTP),PHA(NR1,NTP),AS(NTP)
       DIMENSION LAP(2,64) ,A(64)
C      SET THE PHASE SHIFT OF THE BOTTOM ROW
       NROW=NR1-1
       NN=NBP(NROW)
       DO 1 J=1,NN
       LAP(2,J)=J
    1  AS(J)=PHA(1,1)
       KK=NN
       DO 10 I=1,NROW
       II=NROW-I
       I1=II+1
       NN=NBP(II)
       IF(II.LE.0)NN=1
       DO 12 J=1,NTP
       LAP(1,J)=LAP(2,J)
   12  A(J)=AS(J)
       KN=0
       DO 20 L=1,KK
       LL=LAP(1,L)
       DO 21 N=1,NTP
       IF(MC(I1,N).NE.LL)GO TO 21
       JJ=N
       GO TO 22
   21  CONTINUE
   22  NMD=MOD(JJ,NN)
       IF(NMD.EQ.0)NMD=NN
       DO 30 K=1,NN
       IND=JJ-NMD+K
       IF(NN.EQ.1)IND=JJ
       KN=KN+1
       LAP(2,KN)=IND
   30  AS(IND)=A(LL)+PHA(I1,LL)
   20  CONTINUE
       KK=KN
   10  CONTINUE
       RETURN
       END
```

```
      SUBROUTINE NTWK(NTP,NR1,NBP,NBK,MC,PHA)
C*****THIS SUBROUTINE FINDS THE CONNECTION OF A BUTLER MATRIX OR FFT
C     GIVEN THE NUMBER OF ROWS AND THE NUMBER OF PORTS IN EACH BLOCK IN
C     EACH ROW
C*****COMPILED BY J. K. HSIAO
C*****FIRST VERSION IS COMPILED ON MAY 3,1976
C*****NTP,  NUMBER OF TOTAL INPUT PORTS OR SAMPLES
C*****NROW, NUMBER OF ROWS REQUIRED TO PERFORM THE TRANSFORMATION
C*****NBP, AN ARRAY STORES THE NUMBER OF PORTS IN EACH BLOCK AT EACH
C     ROW. EACH BLOCK IN A ROW HAS THE SAME NUMBER OF PORTS
C*****NBK, AN ARRAY STORES THE NUMBER OF BLOCKS IN EACH ROW.
C*****MC, A TWO DIMENSIONAL ARRAY STORES THE CONNCTIONS OF THE NETWORK.
C     FIRST INDEX OF THE ARRAY REPRESENTS THE NUMBER OF CURRENT ROW. THE
C     LOCATION OF THE SECOND INDEX REPRESENTS THE PHYSICAL LOCATION OF
C     THE PREVIOUS ROW WHILE THE CONTENTS OF IT IS THE CONNECTION TO THE
C     CURRENT ROW
      DIMENSION MC(NR1,NTP),PHA(NR1,NTP)
      DIMENSION NFTS(64),NBK(16),NBP(16)
C     COMPUTES THE NUMBER OF PORTS IN EACH BLOCK
      NROW=NR1-1
      PI=3.1415926536
      PI2=PI*2.
      NBP(NR1)=1
      NTP2=NTP/2
      DO 10 I=1,NR1
   10 NBK(I)=NTP/NBP(I)
C**** NFTS ARRAY STORES THE LOCATION OF THE SAMPLES IN EACH BEAM(OR
C     FREQUENCY SAMPLE). THE STRUCTURE IS CHARACTERIZED BY TWO NUMBERS,
C     NTS,NUMBER OF TIME SAMPLES(OR INPUT PORTS) AND NFS, NUMBER OF
C     FREQUENCY SAMPLES(OR NUMBER OF BEAMS).  FOR EXAMPLE, NFTS((3-1)*
C     NTS+1) IS THE PHYSICAL LOCATION OF THE FIRST TIME SAMPLE IN THE
C     THIRD FREQUENCY GROUP( OR OF THE THIRD BEAM),THIS IS REPRESENTED
C     BY LMC
C
C     SET THE INITIAL NFTS ARRAY
      DO 11 I=1,NTP
   11 NFTS(I)=I
C**** NTS1 IS THE PREVIOUS VALUES OF THE NUMBER OF TIME SAMPLES(OR INPUT
C     PORTS)
C**** NTS2 IS THE CURRENT VALUE
C**** NFS1 IS THE PREVIOUS VALUE OF THE NUMBER OF FREQUENCY SAMPLES(OR
C     BEAMS)
C*****NFS2 IS THE CURRENT VALUE
C
C
C     SET THE INIAL VALUES OF NTS AND NFS
      NTS1=NTP
      NFS1=1
      DO 20 I=1,NR1
C     MM THE NUMBER OF BLOCKS OF THE CURRENT ROW
```

```
C     NN, THE NUMBER OF PORTS IN EACH BLOCK OF THE CURRENT ROW
      MM=NBK(I)
      NN=NBP(I)
C     SET NTS2 AND NFS2
      NTS2=NTS1/NN
      NFS2=NTP/NTS2
C**** THE ACTUAL REQUIRED PHASE GRADIENT BETWEEN SUCCESSIVE ELEMENT FOR
C     THE FIRST BEAM IS
      PAG=PI/NFS2
C***  THAVAILABLE PHASE GRADIENT FOR THE FIRST BEAM IN EACH BLOCK IS
      PSG=PI/NN
      KK=0
      DO 30 J=1,MM
      MODJ=MOD(J,NFS1)
      IF(MODJ.EQ.0)MODJ=NFS1
      JJ=(J-1)/NFS1+1
      PAGG=PAG*(MODJ*2-1)
      PASGD=PSG-PAGG
      DO 30 K=1,NN
      K1=K-1
      KK=KK+1
      LMC=(MODJ-1)*NTS1+(K-1)*NTS2+JJ
      MCLOC=NFTS(LMC)
      MC(I,MCLOC)=KK
      IF(KK.LE.NTP2)GO TO 31
      KKI=NTP-KK+1
      PHA(I,KK)=PHA(I,KKI)
      GO TO 30
   31 IF(PASGD.GT.0.)GO TO 32
      PHA(I,KK)=ABS(PASGD)*(NN-K)
      GO TO 30
   32 PHA(I,KK)=PASGD*K1
   30 CONTINUE
C     RECORDING  THE RREQUENCY SAMPLE OR BEAM POSITION INTO NFTS ARRAY
      NTS1=NTS2
      NFS1=NFS2
      KK=0
C     MNS IS THE NUMBER OF BLOCKS WITHIN EACH GROUP OF FREQUENCY SAMPLES
      MNS=MM/NTS1
      DO 40 J=1,NFS1
      JMOD=MOD(J,MNS)
      IF(JMOD.EQ.0)JMOD=MNS
      JJ=(J-1)/MNS+1
      DO 40 K=1,NTS1
      KK=KK+1
   40 NFTS(KK)=(K-1)*NFS1+(JMOD-1)*NN+JJ
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FRAME(XM,YM,XSL,YSL,SY,HN,NX,NY)
      COMMON/C$1/PLTAY(500)
      YMSY=YM+SY
      HLAB=HN*.035
      HLAS=HLAB*.035
      WLAB=4.*HLAB/7.
      XSCL=XSL/NX
      YSCL=YSL/NY
      DY=YM/NY
      Y=SY
      NNY=NY+1
      CALL PLOT(0.,SY,3)
      CALL PLOT(XM,SY,2)
      CALL PLOT(XM,YMSY,2)
      CALL PLOT(0.,YMSY,2)
      CALL PLOT(0.,SY,2)
      DO 10 I=1,2
      Y=SY
      IF(I.GT.1)GO TO 12
      X1=0.
      X2=-.2
      X3=-.1
      GO TO 13
   12 X1=XM
      X2=XM+.2
      X3=XM+.1
   13 DO 10 J=1,NNY
      CALL PLOT(X1,Y,3)
      MODY=MOD(J-1,10)
      IF(MODY.NE.0)GO TO 11
      CALL PLOT(X2,Y,2)
      IF(I.GT.1)GO TO 10
      A=YSCL*(J-1-NY)
      CALL NUMBER(-6.5*WLAB,Y-HLAB/2.,HLAB,A,0.,4HF3.0)
      GO TO 10
   11 CALL PLOT(X3,Y,2)
   10 Y=Y+DY
      DX=XM/NX
      NXX=NX+1
      DO 20 I=1,2
      X=0.
      IF(I.GT.1)GO TO 22
      Y1=SY
      Y2=Y1-.2
      Y3=Y1-.1
      GO TO 23
   22 Y1=YMSY
      Y2=Y1+.2
      Y3=Y1+.1
   23 DO 20 K=1,NXX
      KK=K-1
      CALL PLOT(X,Y1,3)
      MODX=MOD(KK,10)
      IF(MODX.NE.0)GO TO 21
      CALL PLOT(X,Y2,2)
      IF(I.GT.1)GO TO 20
      A=KK*XSCL
      CALL NUMBER(X-2.5*WLAB,SY-HLAB*3.0,HLAB,A,0.,4HF3.0)
      GO TO 20
   21 CALL PLOT(X,Y3,2)
   20 X=X+DX
```

```
      CALL SYMBOL(.5*XM-17.5*WLAB,-5.*HLAB+SY,HLAS,22HPARAMETER U IN DEG
     CREES,0.,22)
   35 CALL SYMBOL(-7.*WLAB,YM/2.+SY-15.*WLAB,HLAS,18HARRAY PATTERN (DB),
     *90.,18)
 32   CALL PLOT(0.,0.,3)
      END



      SUBROUTINE SIMCX(IS,ORIG,NN,MAT,MCT,ANS,LK)
C     IDENT NUMBER - F1002R00
C     TITLE - COMPLEX MATRIX INVERSION, SOLUTION OF LINEAR EQUATIONS
C     IDENT NAME - F1-NRL-SIMCX
C     LANGUAGE - FORTRAN
C     COMPUTER - CDC-3800
C     CONTRIBUTOR - JANET P. MASON, CODE 7813, RESEARCH COMPUTATION
C                   CENTER, MIS DIVISION
C     ORGANIZATION - NRL - NAVAL RESEARCH LABORATORY - WASHINGTON, D.C.
C                                                                  20390
C     DATE - 16 DECEMBER 1970
C     PURPOSE - TO SOLVE THE COMPLEX MATRIX EQUATION AX=B WHERE A IS A
C               SQUARE COEFFICIENT MATRIX AND B IS A MATRIX OF CONSTANT
C               VECTORS.  THE DETERMINANT AND INVERSE OF A ARE ALSO
C               OBTAINED.
      COMPLEX SUM, MAT,ORIG,ANS,B0,B2,B4,B6,B8,B10,B11,B13,B15,CC,CC2,B3
      EQUIVALENCE(B2,C),(CC,CX),(CC2,CX2)
      DIMENSION MAT(MCT,1),ORIG(NN,1),ANS(MCT),C(2),CX(2),CX2(2)
   10 FORMAT(1X, 2E12.6)
   15 FORMAT(25H  THIS MATRIX IS SINGULAR/)
   18 FORMAT(1H0,' VALUE OF DETERMINANT IS ',2E12.6,//)
 21   FORMAT(1X,2E12.6,5X,2E12.6)
   23 FORMAT(8X,'ORIGINAL CONSTANTS',21X,'DERIVED CONSTANTS'/)
   26 FORMAT(1H1,6X,'THE INVERSE (BY COLUMNS)')
   27 FORMAT(1H0)
   28 FORMAT(1H1,6X,'VALUES OF THE UNKNOWNS')
   35 FORMAT(9X,'IDENTITY MATRIX')
      B3=(-1.0,0.0)
      B4=(0.0,0.0)
      B11=(1.0,0.0)
      ICT=MCT
      JSING=MCT
      MT=MCT+1
      NCT=MCT+MCT
C     PUT ORIGINAL MATRIX INTO MAT
      IF(IS.EQ.0)GO TO 39
      ICT=MCT+IS
      NCT=ICT
 39   DO 2 J=1,ICT
      DO 2 I=1,MCT
      MAT(I,J)=ORIG(I,J)
    2 CONTINUE
      IF(IS.NE.0)GO TO 30
C     PUT IDENTITY MATRIX INTO RIGHT HALF OF MAT
   31 DO 32 J=MT,NCT
      DO 32 I=1,MCT
   32 MAT(I,J)=B4
      DO 33 J=1,MCT
   33 MAT(J,J+MCT)=B3
C     FORM TRIANGULARIZED MATRIX
```

```
   30 JCT=MCT-1
      DO 3 J=1,JCT
      KK=J+1
      GOTO 25
   24 DO 4 K=KK,MCT
      B8=MAT(K,J)/MAT(J,J)
      DO 5 L=J,MCT
      B10=B8*MAT(J,L)
    5 MAT(K,L)=MAT(K,L)-B10
    4 CONTINUE
C     VALUE OF DETERMINANT
    3 B11=B11*MAT(J,J)
      B11=B11*MAT(MCT,MCT)
      LOW=-MCT
      MO=-1
C     TO DO ONE OR MORE BACK SOLUTIONS
      DO 6 MINC=MT,MCT
      JFIN=MCT
      IX=0
C     BACK SOLUTION
      DO 6 INN=LOW,MO
      M=IABS(INN)
      B0=-MAT(M,MINC)
      B2=MAT(M,M)
      B4=(0.0,0.0)
      IF(IX) 7,22,7
   22 IX=IX+1
      GOTO 8
    7 MO2=-JFIN
      DO 9 INN=LOW,MO2
      N=IABS(INN)
    9 B4=B4+MAT(M,N)*MAT(N,MINC)
      B0=B0-B4
      JFIN=JFIN-1
    8 IF(C(1).EQ.0.0.AND.C(2).EQ.0.0)GO TO 13
   29 MAT(M,MINC)=B0/B2
      ANS(M)=B0/B2
    6 CONTINUE
      DO 40 J=MT,MCT
      JJ=J-MCT
      DO 40 I=1,MCT
   40 ORIG(I,JJ)=MAT(I,J)
      IF(LK.GT.0)RETURN
      IF(IS.EQ.0)GO TO 34
      GO TO 41
C     CHECK FOR SINGULARITY AND TO SEE IF FIRST TERM = 0
   25 JV=J
      CC=MAT(J,J)
      IF(CX(1).NE.0.0.OR. CX(2).NE.0.0)GO TO 12
   11 IF(JV.NE.JSING)GO TO 14
```

```
   13 PRINT15
      PRINT 100,J,(MAT(K,J),K=1,MCT)
  100 FORMAT(10X,I5,8F10.4)
      PRINT 21,((MAT(I,J),I=1,MCT),J=1,MCT)
      RETURN
   14 JV=JV+1
      CC2=MAT(JV,J)
      IF(CX2(1).EQ.0.0.AND.CX2(2).EQ.0.0)GO TO 11
   16 DO 17 JJ=J,MCT
      B6=MAT(J,JJ)
      MAT(J,JJ)=MAT(JV,JJ)
   17 MAT(JV,JJ)=B6
      B11=-B11
  12  CONTINUE
      GOTO 24
C     PRINT SUBSTITUTIONS BACK INTO ORIGINAL MATRIX
   45 DO 20 NNV=1,IS
      PRINT 27
   44 PRINT23
      DO 20 LL=1,MCT
      B13=(0.0,0.0)
      DO 19 MM=1,MCT
   19 B13=ORIG(LL,MM)*MAT(MM,MCT+NNV)+B13
      B15=-ORIG(LL,MCT+NNV)
      PRINT21,B15,B13
   20 CONTINUE
      RETURN
C     PRINT TITLE - THE INVERSE
   34 PRINT 26
      GO TO 43
C     PRINT TITLE - VALUES OF UNKNOWNS
   41 PRINT 28
   43 DO 38 JJ=KT,MCT
      PRINT 27
      DO 38 II=1,MCT
   38 PRINT 10, MAT(II,JJ)
C     PRINT VALUE OF DETERMINANT
      PRINT 10,B11
      IF(IS.NE.0)GO TO 45
C     PRINT IDENTITY MATRIX
      PRINT 35
      DO 36 K=1,MCT
      PRINT 27
      DO 36 I=1,MCT
      SUM=(0.0,0.0)
      DO 37 J=1,MCT
   37 SUM=ORIG(K,J)*MAT(J,MCT+I)+SUM
   36 PRINT 10,SUM
      RETURN
      END
```

```
      SUBROUTINE BLK(NM,NN,S11,S12,S21,S22)
      DIMENSION NBP(16),NBK(16)
      DIMENSION MC(8,16),PHA(8,16)
      DIMENSION S11(NM,NM),S12(NM,NM),S21(NM,NM),S22(NM, NM)
      COMMON/C$5/T11(8,8),T12(8,8),T21(8,8),T22(8,8),R11(8,8),R12(8,8),
     C R21(8,8),R22(8,8),SPACE(7168)
      COMPLEX S11,S12,S21,S22
      COMPLEX T11,T12,T21,T22,R11,R12,R21,R22
      IF(NN.GT.2)GO TO 1
      CALL TWOPT(S11,S12,S21,S22,NM)
      RETURN
    1 II=0
      N2=NN
    3 N2=N2/2
      IF(N2.LE.0)GO TO 2
      II=II+1
      GO TO 3
    2 DO 10 I=1,II
   10 NBP(I)=2
      II=II+1
      CALL NTWK(NN,II,NBP,NBK,MC,PHA)
      CALL STRF(NM,NN,II,NBP,NBK,MC,PHA,S11,S12,S21,S22)
      RETURN
      END
```

```
      SUBROUTINE INVS1(NM,NN,S12)
      COMMON/C$5/A1(32),T(32,64),SPACE(4032)
      DIMENSION S12(NM,NM)
      COMPLEX A1,T,S12
      CALL SIMCX(0,S12,NM,T,NN,A1,1)
      RETURN
      END
```

```
      SUBROUTINE INVS2(NM,NN,S12)
      COMMON/C$5/A1(8),T(8,16),SPACE(7920)
      DIMENSION S12(NM,NM)
      COMPLEX A1,T,S12
      CALL SIMCX(0,S12,NM,T,NN,A1,1)
      RETURN
      END
```

```
      SUBROUTINE TWOPT (S11,S12,S21,S22,M)
      DIMENSION S11(M,M),S12(M,M),S21(M,M),S22(M,M)
      COMPLEX S11,S12,S21,S22
      COMMON/C$4/A,B,C,D
      BCD=B+C+D
      IF(BCD.GT.0.)GO TO 1
      AR=10.**(-A*.05)
      IF(A.LE.0.)AR=0.
      A1=SQRT(.5-AR*AR)
      A2=SQRT(.5-AR*AR)
      B1=AR
      B2=AR
      GO TO 2
    1 B1=10.**(-A*.05)
      IF(A.LE.0.)B1=0.
      B2=10.**(-B*.05)
      IF(B.LE.0.)B2=0.
      A1=10.**(-C*.05)
      A2=10.**(-D*.05)
    2 S11(1,1)=B1*CMPLX(0.,-1.)
      S11(2,2)=B1*CMPLX(0.,-1.)
      S22(1,1)=B1*CMPLX(0.,-1.)
      S22(2,2)=B1*CMPLX(0.,-1.)
      S11(1,2)=B2*CMPLX(-1.,0.)
      S11(2,1)=B2*CMPLX(-1.,0.)
      S22(1,2)=B2*CMPLX(-1.,0.)
      S22(2,1)=B2*CMPLX(-1.,0.)
      S12(1,1)=A1*CMPLX(1.,0.)
      S12(2,2)=A1*CMPLX(1.,0.)
      S21(1,1)=A1*CMPLX(1.,0.)
      S21(2,2)=A1*CMPLX(1.,0.)
      S12(1,2)=A2*CMPLX(0.,-1.)
      S12(2,1)=A2*CMPLX(0.,-1.)
      S21(1,2)=A2*CMPLX(0.,-1.)
      S21(2,1)=A2*CMPLX(0.,-1.)
      RETURN
      END
```

```
      SUBROUTINE TRT(NTP,TRFF,LL,NMX)
      COMMON/C$6/APMT(32,32),ANGL(32,32),ANGT(32),TRID(32,32),SPACE(4064
     *)
      DIMENSION TRFF(NMX,NMX)
      COMPLEX TRFF,TRID
      IF(LL.GT.0)GO TO 1
      DO 10 I=1,NTP
      DO 10 J=1,NTP
   10 TRID(I,J)=TRFF(I,J)
      RETURN
    1 DO 20 I=1,NTP
      DO 20 J=1,NTP
   20 TRFF(I,J)=TRID(I,J)
      RETURN
      END
```

```
      SUBROUTINE TRFIDL(NTP)
      COMMON/C$6/AMPT(32,32),ANGL(32,32),ANGT(32),TRFF2(32,32), TR(32,32
     C),SUMR(2016)
      COMPLEX TRFF2,TR
      PI=3.1415926536
      PI2=PI*2.
      RTA=180./PI
      A=SQRT(1./NTP)
      P=-PI/NTP
      DO 10 I=1,NTP
      PP=(I-1)*P
      PR=P*(I-.5)*2.
      DO 10 J=1,NTP
      PP=AMOD(PP,PI2)
      RE=A*COS(PP)
      RI=A*SIN(PP)
      TRFF2(I,J)=CMPLX(RE,RI)
      AMPT(I,J)=A
      ANGL(I,J)=PP*RTA
   10 PP=PP+PR
      RETURN
      END
```

```
      SUBROUTINE PHASAN (TAINC,I)
      COMMON/C$6/CONTA(4096),SINTA(4096)
      PI=3.1415926536
      PI2=PI*2.
      TA=0.
      I=0
    1 I=I+1
      CONTA(I)=COS(TA)
      SINTA(I)=SIN(TA)
      TA=TA+TAINC
      IF(TA.GE.PI2)RETURN
      GO TO 1
      END
```

```
      COMPLEX FUNCTION AR(AUG)
      AMP=1.
      AG=AUG
      RE=AMP*COS(AG)
      RI=AMP*SIN(AG)
      AR=CMPLX(RE,RI)
      RETURN
      END
```

```
      FUNCTION CANG(SR)
      COMPLEX SR
      A1=REAL(SR)
      A2=AIMAG(SR)
      CANG=ATAN2(A2,A1)
      RETURN
      END
```